

Coexistence in a strongly coupled system describing a two-species cooperative model[☆]

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Received 10 August 2006; received in revised form 19 September 2006; accepted 10 November 2006

Abstract

This paper is concerned with a cooperative two-species Lotka–Volterra model. Using the fixed point theorem, the existence results of solutions to a strongly coupled elliptic system with homogeneous Dirichlet boundary conditions are obtained. Our results show that this model possesses at least one coexistence state if the birth rates are big and cross-diffusions are suitably weak.

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Keywords: Ecological model; Coexistence; Strongly coupled; Diffusion; Upper and lower solutions

1. Introduction

A lot of work has been done to two-species Lotka–Volterra models and the permanence and global stability of systems have been given in the literature (see for example, [3]). May [9] proposed the two-species cooperating model in 1976 as the following:

$$\begin{cases} \frac{du}{dt} = a_1 u \left(1 - \frac{u}{K_1 + b_1 v} - c_1 u \right), \\ \frac{dv}{dt} = a_2 v \left(1 - \frac{v}{K_2 + b_2 u} - c_2 v \right). \end{cases} \quad (1.1)$$

In biological terms, u and v stand for the spatial densities of each of the species, respectively. The constant a_i is its respective net birth rate, while the coefficients $a_1 c_1$ and $a_2 c_2$ are intra-specific competitions. We have assumed logistic growth for one species with carrying capacity $K_1 + b_1 v$ and the other species with carrying capacity $K_2 + b_2 u$. Here the parameters $a_i, b_i, c_i, K_i, (i = 1, 2)$ are all positive constants. There are many results about this ODE system. In this paper, we will consider the strongly coupled system with Dirichlet boundary condition:

[☆] The work is partially supported by the NSF of Jiangsu Province (BK2006064) and also by the NSF of Jiangsu Education Committee (05KJB110154).

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$$\begin{cases} -\Delta \left[\left(d_1 + \alpha_1 u + \frac{\gamma_1}{\beta_1 + v} \right) u \right] = a_1 u \left(1 - \frac{u}{K_1 + b_1 v} - c_1 u \right), & x \in \Omega, \\ -\Delta \left[\left(d_2 + \frac{\gamma_2}{\beta_2 + u} + \alpha_2 v \right) v \right] = a_2 v \left(1 - \frac{v}{K_2 + b_2 u} - c_2 v \right), & x \in \Omega, \\ u(x) = v(x) = 0, & x \in \partial\Omega, \end{cases} \quad (1.2)$$

where the diffusion term of the first equation in (1.2) can be written as

$$\operatorname{div} \left\{ \left(d_1 + 2\alpha_1 u + \frac{\gamma_1}{\beta_1 + v} \right) \nabla u - \frac{\gamma_1 u}{(\beta_1 + v)^2} \nabla v \right\}.$$

The term $d_1 + 2\alpha_1 u + \frac{\gamma_1}{\beta_1 + v}$ represents the “self-diffusion”, which means that the main cause of dispersion of the population is the random motion, while the term $-\frac{\gamma_1 u}{(\beta_1 + v)^2}$ represents the “self-diffusion”, which models the interaction between individuals and here implies that the chaseable capacity of the species u is decreasing with the enhanced resistance of the species v , the sign “−” means that the flux of u is directed toward increasing population of v , i.e. the two cooperative species chase each other. Here the boundary conditions imply that the habitat Ω is surrounded by hostile environment, a solution (u, v) to the system (1.2) is said to be positive if $u(x) > 0$ and $v(x) > 0$ for all $x \in \Omega$, the existence of a positive solution (u, v) to (1.2) is also called a coexistence.

The system with “cross-diffusion” was proposed by Shigesada, Kawasaki and Teramoto [15] to describe a ecological model. The strongly coupled elliptic equations have received considerable attention in recent years, and various forms of the systems have been considered in the literature (see for example, [1,4,6,7,13,16]). In particular, Leung and Fan in [6] considered the following elliptic systems

$$\begin{cases} -\Delta \phi(u) = f(x, u, v), & x \in \Omega, \quad u = 0, \quad x \in \partial\Omega, \\ -\Delta \psi(v) = g(x, u, v), & x \in \Omega, \quad v = 0, \quad x \in \partial\Omega \end{cases}$$

and found a positive solution under some conditions of f and g .

Ruan [12] considered the system with cross-diffusion pressures which are linear with respect to the densities,

$$\begin{cases} -\Delta[(d_1 + \alpha_{11}u_1 + \alpha_{12}u_2)u_1] = u_1(a_1 - b_{11}u_1 - b_{12}u_2), & x \in \Omega, \\ -\Delta[(d_2 + \alpha_{21}u_1 + \alpha_{22}u_2)u_2] = u_2(a_2 - b_{21}u_1 - b_{22}u_2), & x \in \Omega, \end{cases}$$

positive solutions under homogeneous Dirichlet conditions are found when the net birth rates a_1 and a_2 lie in a certain range, or if cross-diffusions are sufficiently large. In [7,8], the system with homogeneous Neumann conditions is discussed. They studied the effects of diffusion, self-diffusion and cross-diffusion and showed that there is no non-constant solution if diffusion or self-diffusion is strong, or if cross-diffusion is weak while non-constant solutions do exist if cross-diffusion is suitably strong.

Ryu and Ahn [13,14] investigated the existence of positive solutions for a more general system of two equations which are density dependent diffusions and showed the existence of positive solutions by the method of fixed point indexes. The existence of positive solutions to the systems with more general boundary conditions was given. Recently, Kim and Lin [5] discussed the three species of a strongly coupled elliptic system and gave the sufficient condition for the coexistence by Schauder fixed point theorem. Chen and Peng [2] considered a strongly coupled prey–predator model, and discussed the existence and uniqueness of coexistence states using bifurcation theory. Pao [10] gave the method of constructing of solutions for a general class of strongly coupled elliptic systems by the method of upper and lower solutions and its associated monotone iterations.

Motivated by these results, we are interested in the investigation of the existence of positive solutions for (1.2). A sufficient condition for the coexistence will be given by rewriting the strongly coupled systems in the equivalent weakly coupled equations.

2. Coexistence

As in [10], we first give a equivalent form of the problem (1.2):

$$\begin{cases} -\Delta[D_1(u, v)] = f_1(u, v), & x \in \Omega, \\ -\Delta[D_2(u, v)] = f_2(u, v), & x \in \Omega, \\ u(x) = 0, \quad v(x) = 0, & x \in \partial\Omega, \end{cases} \quad (2.1)$$

where

$$\begin{aligned} D_1(u, v) &= \left(d_1 + \alpha_1 u + \frac{\gamma_1}{\beta_1 + v} \right) u, & f_1 &= a_1 u \left(1 - \frac{u}{K_1 + b_1 v} - c_1 u \right), \\ D_2(u, v) &= \left(d_2 + \frac{\gamma_2}{\beta_2 + u} + \alpha_2 v \right) v, & f_2 &= a_2 v \left(1 - \frac{v}{K_2 + b_2 u} - c_2 v \right). \end{aligned}$$

Define

$$w = D_1(u, v), \quad z = D_2(u, v).$$

Since the Jacobian J of the transformation w, z is given by

$$\begin{aligned} J &= \frac{\partial(w, z)}{\partial(u, v)} = \left(d_1 + 2\alpha_1 u + \frac{\gamma_1}{\beta_1 + v} \right) \left(d_2 + \frac{\gamma_2}{\beta_2 + u} + 2\alpha_2 v \right) v - \frac{\gamma_1 u}{(\beta_1 + v)^2} \frac{\gamma_2 v}{(\beta_2 + u)^2} \\ &\geq d_1 d_2 \quad \text{for } (u, v) \geq (0, 0). \end{aligned}$$

The inverse $u = g_1(w, z)$, $v = g_2(w, z)$ exist whenever $(u, v) \geq (0, 0)$. Then the corresponding equivalent (1.2) becomes

$$\begin{cases} -\Delta w + k_1 w = F_1(u, v), & x \in \Omega, \\ -\Delta z + k_2 z = F_2(u, v), & x \in \Omega, \\ u = g_1(w, z), \quad v = g_2(w, z), & x \in \Omega, \\ w(x) = 0, \quad z(x) = 0, & x \in \partial\Omega, \end{cases} \quad (2.2)$$

where $F_i(u, v) = k_i D_i(u, v) + f_i(u, v)$, $i = 1, 2$.

Now we assume that

$$k_1 \geq \frac{a_1(1 + c_1 K_1)}{K_1 \alpha_1}, \quad k_2 \geq \frac{a_2(1 + c_2 K_2)}{K_2 \alpha_2}.$$

Then

$$\begin{aligned} \frac{\partial F_1}{\partial u} &= k_1 d_1 + a_1 + 2 \left(k_1 \alpha_1 - a_1 c_1 - \frac{a_1}{K_1 + b_1 v} \right) u + k_1 \frac{\gamma_1}{\beta_1 + v} \geq 0, \\ \frac{\partial F_2}{\partial v} &= k_2 d_2 + a_2 + \left(k_2 \alpha_2 - a_2 c_2 - \frac{a_2}{K_2 + b_2 u} \right) v + k_2 \frac{\gamma_2}{\beta_2 + u} \geq 0, \\ \frac{\partial F_1}{\partial v} &= -\frac{K_1 \gamma_1 u}{(\beta_1 + v)^2} + \frac{b_1 a_1 u^2}{(K_1 + b_1 v)^2}, \\ \frac{\partial F_2}{\partial u} &= -\frac{K_2 \gamma_2 v}{(\beta_2 + u)^2} + \frac{b_2 a_2 v^2}{(K_2 + b_2 u)^2}. \end{aligned}$$

Therefore the function $F_1(u, v)$ is nondecreasing in u and $F_2(u, v)$ is nondecreasing in v . But here we find that it is not easy to decide the signs of $\frac{\partial F_1}{\partial v}$ and $\frac{\partial F_2}{\partial u}$.

On the other hand, an elementary calculation gives

$$\begin{aligned} \frac{\partial u}{\partial w} &= \left(d_2 + \frac{\gamma_2}{\beta_2 + u} + 2\alpha_2 v \right) / J, & \frac{\partial v}{\partial w} &= \left(\frac{\gamma_2 v}{(\beta_2 + u)^2} \right) / J, \\ \frac{\partial v}{\partial z} &= \left(d_1 + 2\alpha_1 u + \frac{\gamma_1}{\beta_1 + v} \right) / J, & \frac{\partial u}{\partial z} &= \left(\frac{\gamma_1 u}{(\beta_1 + v)^2} \right) / J. \end{aligned}$$

This shows that $u = g_i(w, z)$ ($i = 1, 2$) are nondecreasing in w and z , respectively, for all $(w, z) \geq (0, 0)$.

Next we give the definition of coupled upper and lower solutions as the following:

Definition 2.1. A pair of 4-vector functions $(\tilde{\mathbf{u}}, \tilde{\mathbf{w}}) = (\tilde{u}, \tilde{v}, \tilde{w}, \tilde{z})$, $(\hat{\mathbf{u}}, \hat{\mathbf{w}}) = (\hat{u}, \hat{v}, \hat{w}, \hat{z})$ in $C^2(\Omega) \cap C(\overline{\Omega})$ are coupled upper and lower solutions of (2.2), if $(\tilde{\mathbf{u}}, \tilde{\mathbf{w}}) \geq (\hat{\mathbf{u}}, \hat{\mathbf{w}})$ and if their components satisfy the relation

$$\begin{cases} -\Delta \tilde{w} + k_1 \tilde{w} \geq F_1(u, v), & -\Delta \hat{w} + k_1 \hat{w} \leq F_1(u, v), & \hat{u} \leq u \leq \tilde{u}, \hat{v} \leq v \leq \tilde{v}, \\ -\Delta \tilde{z} + k_2 \tilde{z} \geq F_2(u, v), & -\Delta \hat{z} + k_2 \hat{z} \leq F_2(u, v), & \hat{u} \leq u \leq \tilde{u}, \hat{v} \leq v \leq \tilde{v}, \\ \tilde{u} \geq g_1(\tilde{w}, \tilde{z}), & \hat{u} \leq g_1(\tilde{w}, \tilde{z}), & x \in \Omega, \\ \tilde{v} \geq g_2(\tilde{w}, \tilde{z}), & \hat{v} \leq g_2(\tilde{w}, \tilde{z}), & x \in \Omega, \\ \tilde{w}(x) \geq 0 = \hat{w}(x), & \tilde{z}(x) \geq 0 = \hat{z}(x), & x \in \partial\Omega. \end{cases} \quad (2.3)$$

We set

$$S = \{\mathbf{u} \in C^\alpha(\overline{\Omega}); \hat{\mathbf{u}} \leq \mathbf{u} \leq \tilde{\mathbf{u}}\}; \quad S^* = \{\mathbf{w} \in C^\alpha(\overline{\Omega}); \hat{\mathbf{w}} \leq \mathbf{w} \leq \tilde{\mathbf{w}}\}$$

where $\mathbf{u} = (u, v)$, $\mathbf{w} = (w, z)$, $\tilde{\mathbf{u}} = (\tilde{u}, \tilde{v})$, $\hat{\mathbf{u}} = (\hat{u}, \hat{v})$ and $\tilde{\mathbf{w}} = (\tilde{w}, \tilde{z})$, $\hat{\mathbf{w}} = (\hat{w}, \hat{z})$.

For definiteness, we choose

$$\tilde{u} = g_1(\tilde{w}, \tilde{z}), \quad \tilde{v} = g_2(\tilde{w}, \tilde{z}), \quad \hat{u} = g_1(\hat{w}, \hat{z}), \quad \hat{v} = g_2(\hat{w}, \hat{z}),$$

which is equivalent to

$$\tilde{w} = D_1(\tilde{u}, \tilde{v}), \quad \tilde{z} = D_2(\tilde{u}, \tilde{v}), \quad \hat{w} = D_1(\hat{u}, \hat{v}), \quad \hat{z} = D_2(\hat{u}, \hat{v}).$$

Then the requirements of (\tilde{u}, \tilde{v}) , (\hat{u}, \hat{v}) in (2.3) are satisfied and those of (\tilde{w}, \tilde{z}) , (\hat{w}, \hat{z}) are reduced to

$$\begin{cases} -\Delta[D_1(\tilde{u}, \tilde{v})] + K_1 D_1(\tilde{u}, \tilde{v}) \geq F_1(u, v), & \hat{u} \leq u \leq \tilde{u}, \hat{v} \leq v \leq \tilde{v}, \\ -\Delta[D_2(\tilde{u}, \tilde{v})] + K_2 D_2(\tilde{u}, \tilde{v}) \geq F_2(u, v), & \hat{u} \leq u \leq \tilde{u}, \hat{v} \leq v \leq \tilde{v}, \\ -\Delta[D_1(\hat{u}, \hat{v})] + K_1 D_1(\hat{u}, \hat{v}) \leq F_1(u, v), & \hat{u} \leq u \leq \tilde{u}, \hat{v} \leq v \leq \tilde{v}, \\ -\Delta[D_2(\hat{u}, \hat{v})] + K_2 D_2(\hat{u}, \hat{v}) \leq F_2(u, v), & \hat{u} \leq u \leq \tilde{u}, \hat{v} \leq v \leq \tilde{v}, \\ \tilde{u}(x) \geq 0 = \hat{u}(x), & \tilde{v}(x) \geq 0 = \hat{v}(x), & x \in \partial\Omega. \end{cases} \quad (2.4)$$

We call the pair (\tilde{u}, \tilde{v}) , (\hat{u}, \hat{v}) satisfying (2.4) and $(\tilde{u}, \tilde{v}) \geq (\hat{u}, \hat{v})$ coupled upper and lower solutions of (1.2).

Now we seek a pair of coupled upper and lower solutions of (1.2) in the form

$$(\tilde{u}, \tilde{v}) = (M_1, M_2), \quad (\hat{u}, \hat{v}) = (g_1(\delta_1 \phi, \delta_2 \phi), g_2(\delta_1 \phi, \delta_2 \phi)),$$

where M_i and δ_i ($i = 1, 2$) are some positive constants with δ_i sufficiently small, and $\phi \equiv \phi(x)$ is the (normalized) positive eigenfunction corresponding to λ_0 , where λ_0 is the smallest eigenvalue of the Laplacian $(-\Delta)$ under Dirichlet boundary condition. It is easy to see that (M_1, M_2) , $(g_1(\delta_1 \phi, \delta_2 \phi), g_2(\delta_1 \phi, \delta_2 \phi))$ satisfy the inequalities in (2.4) if

$$\begin{cases} -\Delta \left[\left(d_1 + \alpha_1 M_1 + \frac{\gamma_1}{\beta_1 + M_2} \right) M_1 \right] \geq k_1 \left(\frac{\gamma_1}{\beta_1} - \frac{\gamma_1}{\beta_1 + M_2} \right) M_1 + a_1 M_1 \left(1 - \frac{M_1}{K_1 + b_1 M_2} - c_1 M_1 \right), \\ -\Delta \left[\left(d_2 + \frac{\gamma_2}{\beta_2 + M_1} + \alpha_2 M_2 \right) M_2 \right] \geq k_2 \left(\frac{\gamma_2}{\beta_2} - \frac{\gamma_2}{\beta_2 + M_1} \right) M_2 + a_2 M_2 \left(1 - \frac{M_2}{K_2 + b_2 M_1} - c_2 M_2 \right), \end{cases} \quad (2.5)$$

$$\begin{cases} \lambda_0 \delta_1 \phi \leq \delta_1 \phi \left[k_1 \left(\frac{\gamma_1}{\beta_1 + M_2} - \frac{\gamma_1}{\beta_1} \right) + a_1 \left(1 - \frac{\hat{u}}{K_1} - c_1 \hat{u} \right) \right] / \left(d_1 + \alpha_1 \hat{u} + \frac{\gamma_1}{\beta_1 + \hat{v}} \right), \\ \lambda_0 \delta_2 \phi \leq \delta_2 \phi \left[k_2 \left(\frac{\gamma_2}{\beta_2 + M_1} - \frac{\gamma_2}{\beta_2} \right) + a_2 \left(1 - \frac{\hat{v}}{K_2} - c_2 \hat{v} \right) \right] / \left(d_2 + \frac{\gamma_2}{\beta_2 + \hat{u}} + \alpha_2 \hat{v} \right). \end{cases} \quad (2.6)$$

Indeed (M_1, M_2) , $(\delta_1 \phi, \delta_2 \phi)$ satisfy the inequalities in (2.5) if

$$\begin{cases} 0 \geq k_1 \left(\frac{\gamma_1}{\beta_1} - \frac{\gamma_1}{\beta_1 + M_2} \right) + a_1 (1 - c_1 M_1), \\ 0 \geq k_2 \left(\frac{\gamma_2}{\beta_2} - \frac{\gamma_2}{\beta_2 + M_1} \right) + a_2 (1 - c_2 M_2). \end{cases} \quad (2.7)$$

On the other hand, we notice that the relation

$$\delta_1 \phi = \left(d_1 + \alpha_1 \hat{u} + \frac{\gamma_1}{\beta_1 + \hat{v}} \right) \hat{u}, \quad \delta_2 \phi = \left(d_2 + \frac{\gamma_2}{\beta_2 + \hat{u}} + \alpha_2 \hat{v} \right) \hat{v}$$

implies that $0 < \hat{u}d_1 \leq \delta_1 \phi$, $0 < \hat{v}d_2 \leq \delta_2 \phi$, so the inequalities in (2.6) hold if δ_1, δ_2 are sufficiently small and

$$\begin{cases} \lambda_0 \left(d_1 + \frac{\gamma_1}{\beta_1} \right) + k_1 \left(\frac{\gamma_1}{\beta_1} - \frac{\gamma_1}{\beta_1 + M_2} \right) < a_1, \\ \lambda_0 \left(d_2 + \frac{\gamma_2}{\beta_2} \right) + k_2 \left(\frac{\gamma_2}{\beta_2} - \frac{\gamma_2}{\beta_2 + M_1} \right) < a_2. \end{cases} \quad (2.8)$$

Choose $k_1 = \frac{a_1(1+c_1K_1)}{K_1\alpha_1}$, $k_2 = \frac{a_2(1+c_2K_2)}{K_2\alpha_2}$. Assume that

$$\lambda_0 \left(d_1 + \frac{\gamma_1}{\beta_1} \right) + \frac{a_1\gamma_1(1+C_1K_1)}{\alpha_1\beta_1K_1} < a_1, \quad \lambda_0 \left(d_2 + \frac{\gamma_2}{\beta_2} \right) + \frac{a_2\gamma_2(1+C_2K_2)}{\alpha_2\beta_2K_2} < a_2. \quad (2.9)$$

Then the requirement in (2.7) and (2.8) are fulfilled by some constants M_1, M_2 satisfying

$$a_1C_1M_1 \geq a_1 + \frac{a_1\gamma_1(1+c_1K_1)}{K_1\alpha_1\beta_1}, \quad a_2C_2M_2 \geq a_2 + \frac{a_2\gamma_2(1+c_2K_2)}{K_2\alpha_2\beta_2}.$$

Hence under the condition (2.9), there exist positive constants M_i, δ_i ($i = 1, 2$) and ϕ such that the pair (M_1, M_2) , $(g_1(\delta_1\phi, \delta_2\phi), g_2(\delta_1\phi, \delta_2\phi))$ are coupled upper and lower solutions of (1.2).

According to Theorem 2.1 of [10], we have the existence result:

Theorem 2.1. *Under the assumption (2.9) the problem (2.2) admits at least one solution $(\mathbf{u}, \mathbf{w}) \in S \times S^*$ that satisfies $(\bar{\mathbf{u}}, \bar{\mathbf{w}}) \geq (\mathbf{u}, \mathbf{w}) \geq (\hat{\mathbf{u}}, \hat{\mathbf{w}})$. Moreover $\mathbf{u} = (u, v)$ is a solution of (1.2).*

Remark 2.1. It is easy to see that if $\lambda_0(d_1 + \frac{\gamma_1}{\beta_1}) \geq a_1$ or $\lambda_0(d_2 + \frac{\gamma_2}{\beta_2}) \geq a_2$, then (1.2) has no positive solution, see [11] or [12]. Our result shows that if $\lambda_0d_1 < a_1$ and $\lambda_0d_2 < a_2$, then (1.2) has at least one coexistence state provided that cross diffusions γ_1 and γ_2 are sufficiently small.

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